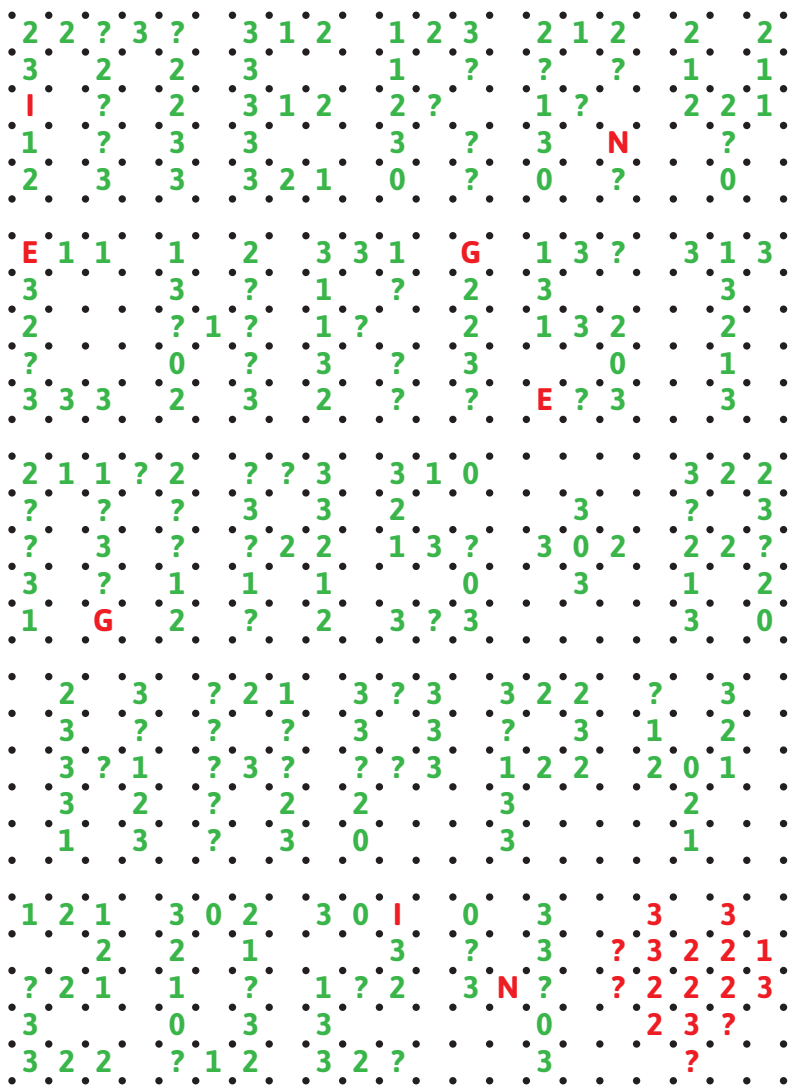
Christmas puzzle 2023



Solution booklet

Dear puzzler

Five years and going! I did the first of these cards in 2019, while on the mend from a bona fide burnout. I spent most of the year doing almost nothing (not by choice mind you) and then, on a whim, sent a home made card to a small group of friends - a folded piece of paper that I had quickly put together in november. It had a definite “graphic design is my passion” vibe to it, and I kinda cringe when I look at it now. I had no plans to make this a recurring thing and definitely wasn’t planning to put in much more effort if I ever did it again.

Then the pandemic hit, early 2020, just when things started to get back into motion. I don’t need to explain what *that* year was like... It did allow things to become a bit more elaborate on the Christmas card side, though. And I’m not exaggerating when I say these first two puzzle card projects, or rather the promise of a *tangible result* they represented, were instrumental in my recovery.

They helped me regain motivation, and relearn to dedicate time to a project that would not immediately see a result, a thing that had become almost alien to me. Now, with the fifth card behind me, I feel that I’m on top of things again. In the process I realised that to create something, even if it’s just a trifle like a Christmas card, is so worth the time, such a satisfying activity!

So it has become unlikely that I’ll stop doing this, even though this one took me much more time than usual. You see, the version you received, was version four. Three other ideas did not pan out, despite a lot of effort. I won’t spoil them, I’m still hopeful I will get them done for a future card. And perhaps it was for the best, because I ended up with a card I’m quite proud of.

Slitherlink, this year’s cover puzzle, is an enduring favourite of mine. I’ve solved countless, but never actually sat down to create one until now. With a twist, of course! I went through several versions, but the things that never changed were (1) a second, smaller puzzle should yield some of the clues, and (2) the number clues should form a legible message.

It soon became clear that that meant I would have too many clues for an interesting challenge, but I was loath to let go of the idea. When I stumbled onto the idea of replacing unnecessary clues with question marks though, it all clicked. And a good thing too, because I had about a week left to finish it and send the files to the printer! That week I went through the puzzle numerous times, each time solving it from a different starting point, and shaving off a few more clues. When the smoke cleared, I had solved it over 20 times and was pretty sick of the damn thing.

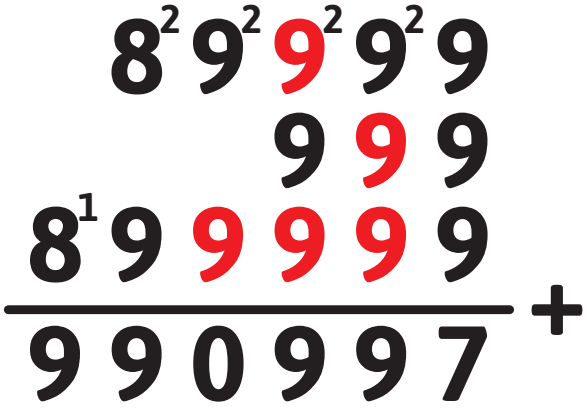
But the feedback has been pretty positive. That means a lot. Thank you.

As always though, solving the puzzle is *never* an expectation! But since you’re reading this, I guess you did it or at least tried. I hope you succeeded, and more importantly, enjoyed it.

Here’s one way to solve it.

Step one. Solve the alphametic.

You can try to solve the slitherlink first, but you'll get much more stuck than if you just start with this one. The puzzle is small enough that you could brute force it; just loop over all possibilities. And I won't blame you for doing that, it's how I created it too. Alphametics that are purely solvable with logic exist, but they are obviously harder to make. It was more important to me to be able to have an appropriate message. Plus, I was kinda running out of time... There are a total of 6 valid solutions, but since you need a solution that is also usable on the front, E, G, I, and N need to be chosen from 0, 1, 2, or 3. This turns out to narrow the options to precisely 1, and we will use this in our deductions.

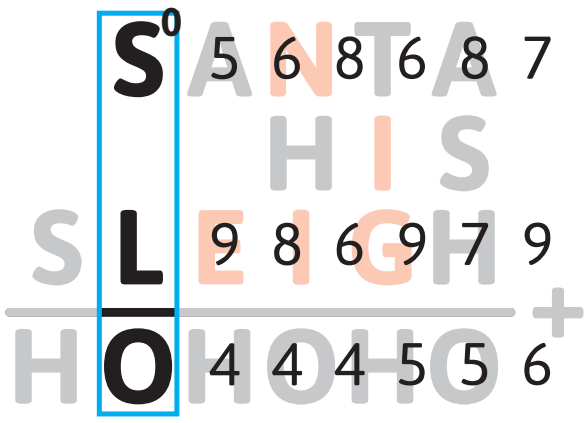


We can often deduce useful clues from the ones column as well as the most significant digits. In this case, let's start on the left side.

The sum has the same number of digits as the largest summand, and since different letters are different digits, this means S can't be 9.

Can S and H differ by more than 1? No. To see this, pretend all other digits are 9's - then at best S+L = 17, and with a maximal carry of 2 that would still only be 19. So S+1 = H.

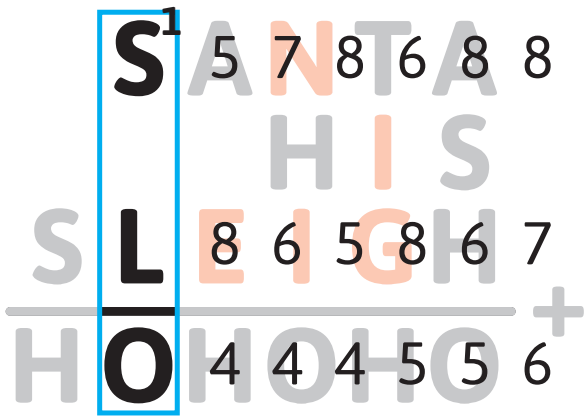
Of course these are overestimations - all digits must be different, so the actual sums will be lower. Remember, EGIN must be chosen from 0123, and AHLOST from 456789.

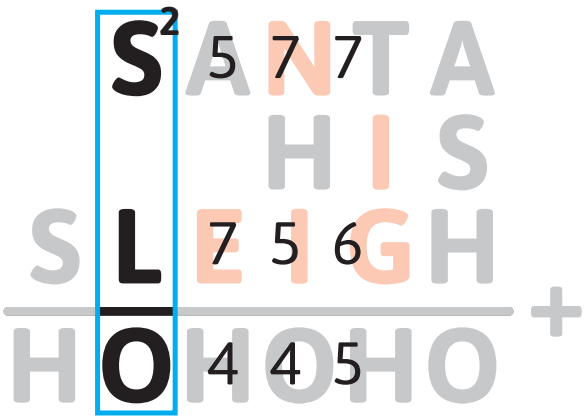


Let's try and use this in column 2. We'll assume for now we don't get a carry from column 3. First we try O = 4 (so S + L = 14). For S and L we get either 5+9 or 6+8 or 8+6, neither can be 7. If we try O = 5, for S and L we'll get 6+9 or 8+7 (7+8 would lead to H = L). O = 6 leaves only S = 7 and L = 9. Every other combination would lead to H = L or H = O, which isn't allowed. Therefore, O = 456 and we have six options for S and L.

What if there is a carry? A nonzero carry will shift things around a bit.

O =7 or O = 8 won't work out, so O is still limited to 456, but we do get new possibilities for S and L:





Now let's check the rightmost column, assuming a carry of 0 from column 3. We only need to consider choices for A and S, since H is implied by S and O can be calculated.

For A = 4, 5, or 6 we have no options for S because all would lead to illegal combinations. For example, A = 5 and S = 8 yields an O of 2, which is outside its range. If A = 7, S is either 4 or 8. However, if SH = 45, O will be 6, and we already know that if O = 6, S must be 7, and that's a contradiction. For A = 8 there are again no valid options. Finally if A = 9, we get S =7, but again the value for O disqualifies this - after all we found that O = 4 means S can't be 7.

We are left with a single option - ASHO = 7894.

Now let's try a carry of 1 from column 3.

Since the math in the rightmost column doesn't depend on that carry we obtain the same three possibilities as before. ASHO = 7456 is invalid again and for a similar reason - O = 6 now implies S must be 8. ASHO = 7894 remains valid. This time around ASHO = 9784 is also ok, because O = 4 now allows for S being 7.

Finally we examine a carry of 2 from column 3.

This time a value of 6 is simply not allowed for O, so again ASHO = 7456 is invalid. ASH = 789 will yield O = 4 and then S can only be 5 or 7 which is a contradiction, so A can't be 7 at all. Only the third option, ASHO = 9784, works.

So we have two possible valid solutions, depending on the actual carry from column 3 - either ASHO = 7894, or ASHO = 9784. That seems eminently doable. All are shown here with the other values they imply (L follows from column 2 and T will be the only remaining digit above 3):

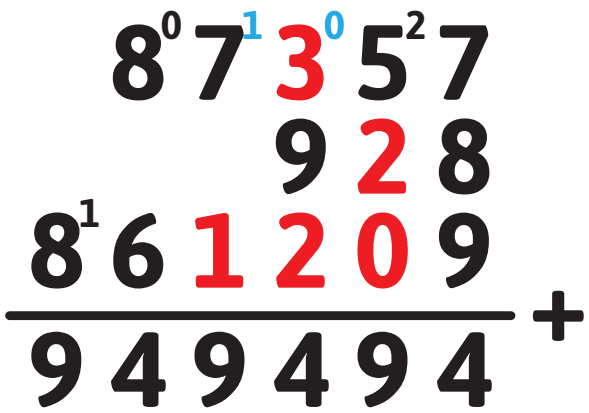
$$\begin{array}{r} 8^{0}\,7\,N\,5^{2}\,7 \\ 9\;I\;8 \\ 8^{1}\,6\,E\,I\,G\,9 \\ \hline 9\,4\,9\,4\,9\,4 \end{array}+ \qquad \begin{array}{r} 8^{1}\,7\,N\,6^{2}\,7 \\ 9\;I\;8 \\ 8^{1}\,5\,E\,I\,G\,9 \\ \hline 9\,4\,9\,4\,9\,4 \end{array}+$$

$$\begin{array}{r} 7^{1}\,9\,N\,5^{2}\,9 \\ 8\;I\;7 \\ 7^{1}\,6\,E\,I\,G\,8 \\ \hline 8\,4\,8\,4\,8\,4 \end{array}+ \qquad \begin{array}{r} 7^{2}\,9\,N\,6^{2}\,9 \\ 8\;I\;7 \\ 7^{1}\,5\,E\,I\,G\,8 \\ \hline 8\,4\,8\,4\,8\,4 \end{array}+$$

The blue circles show the problem spots that disqualify 3 of these.

The central question here is, can we achieve the carry for column 2 with the digits in column 3? For the first diagram, with carry 0, the answer is yes - for example E = 2 will fit, provided column 3 gets no carry from 4.

However for the other three diagrams we have a problem - we must achieve a nonzero carry and the highest value we can choose for E is 3. It's clearly not in the cards here, at least not with a value of 8 or 9 below the line as well...



So diagram 1 survives. Let's start with column 5. We start with 7 and need to get to 9, therefore I+G must be 2, since it is impossible to make them add up to 12. This means column 4 gets no carry, and N+I must be 5. This in turn yields a carry of 1 for column 3, resulting in a value of 1 for E.

Since N and I use up the 2 and 3, G is then 0, which resolves the value of I to 2. N turns out to be 3.

Step two. Solve the slitherlink

If you started with the slitherlink, you'll have discovered the letters occupy key positions in the puzzle, and you can't progress without knowing them. Let's put them in the grid and clean it up.

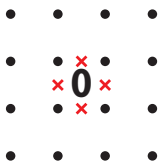
In this first diagram, the letters have been replaced with their digits and the question marks have been removed to arrive at the puzzle proper.

You can see there are some open areas, but none are bigger than 3x5, which I think is about the limit of what you can leave clueless before the puzzle becomes unsolvable.

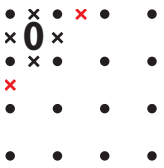
So how to break into this thing? If you've never done a slitherlink puzzle before, at this point the only obvious points of entry are probably the 0 clues, which can't have lines around them (we traditionally mark such places with a little x). But there are a few more rules that can immediately be applied at the start, based on very simple standard patterns.

These patterns occur in almost every slitherlink. It's possible to set a puzzle that has none of them, I did this once for a puzzle hunt (which is a story for another time), but the resulting puzzle will be very hard, uninteresting, or both.

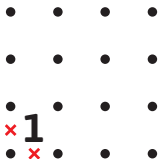
Let's start with an overview of these rules and then apply them all at once, because they will allow us to easily break into the puzzle in a variety of locations. (Newly placed elements are red).



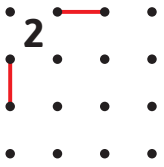
0) A 0 must be surrounded with x's. Is this really a rule? It's kind of a no brainer if you ask me.



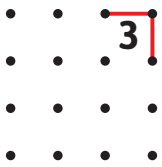
1) A 0 in a corner must additionally have x's on the edges leading out of it along the puzzle edge.



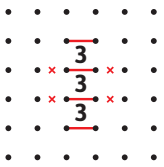
2) A 1 in a corner must have x's on the corner edges.



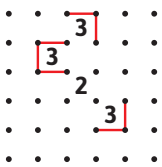
3) A 2 in a corner must have lines on the edges leading out of it along the puzzle edge.



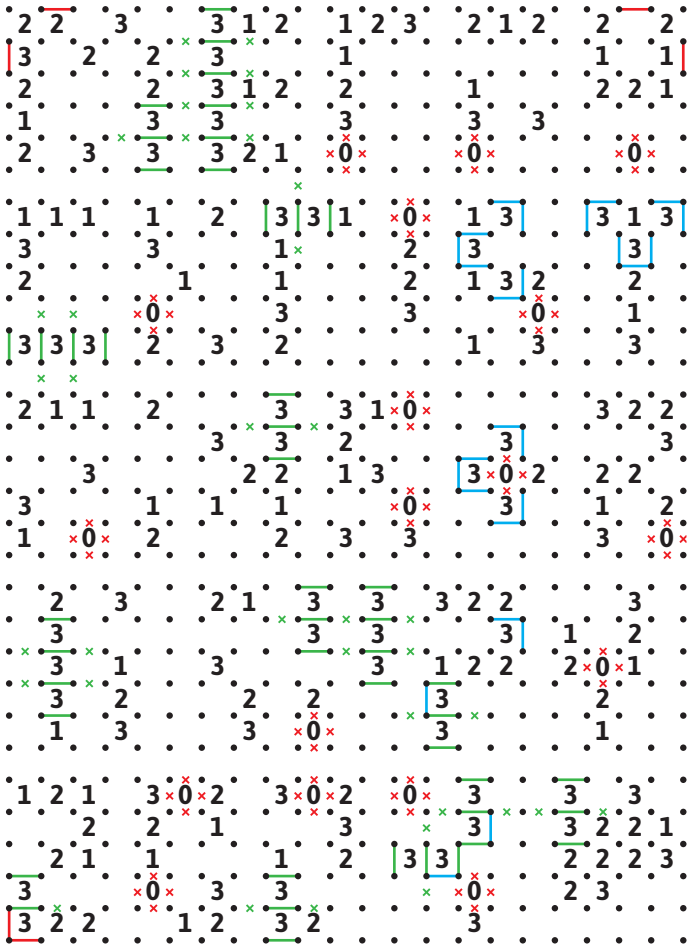
4) A 3 in a corner must have lines on the corner edges.



5) Adjacent 3's in a row or column must be separated by lines with x's on either side, and the row ends must also be lines.



6) 3's on a diagonal (possibly with 2's between them) must have lines on their outer corner.



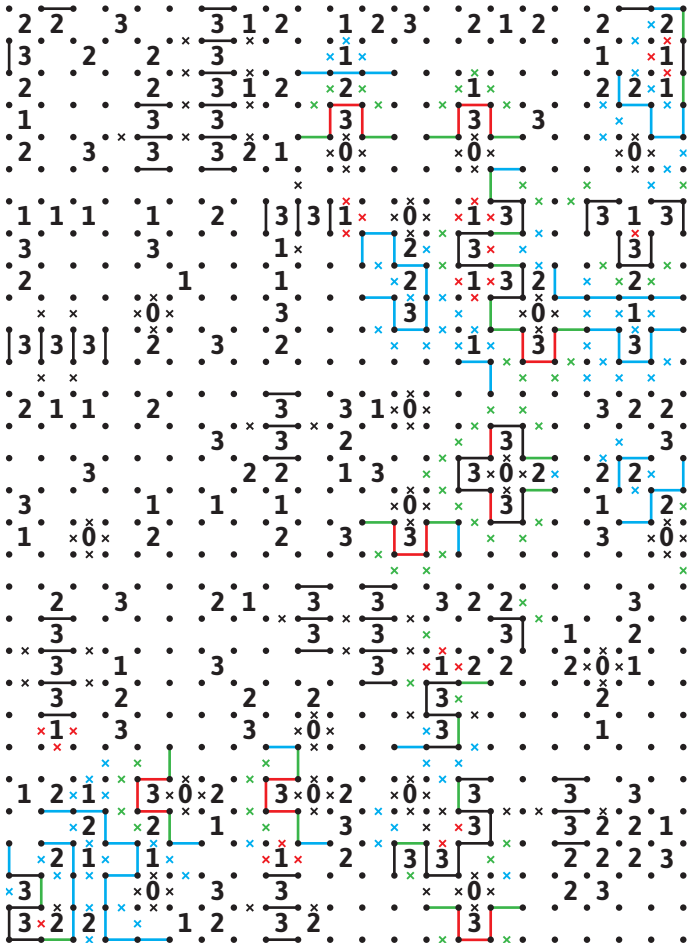
Here is the result of applying these rules everywhere in the puzzle. It's a lot to process, but hopefully the colors help.

Rules 1 and 2 do not apply here.

Rules 0, 3, and 4 are shown in red.

Rule 5 is shown in green.

Rule 6 is shown in blue.

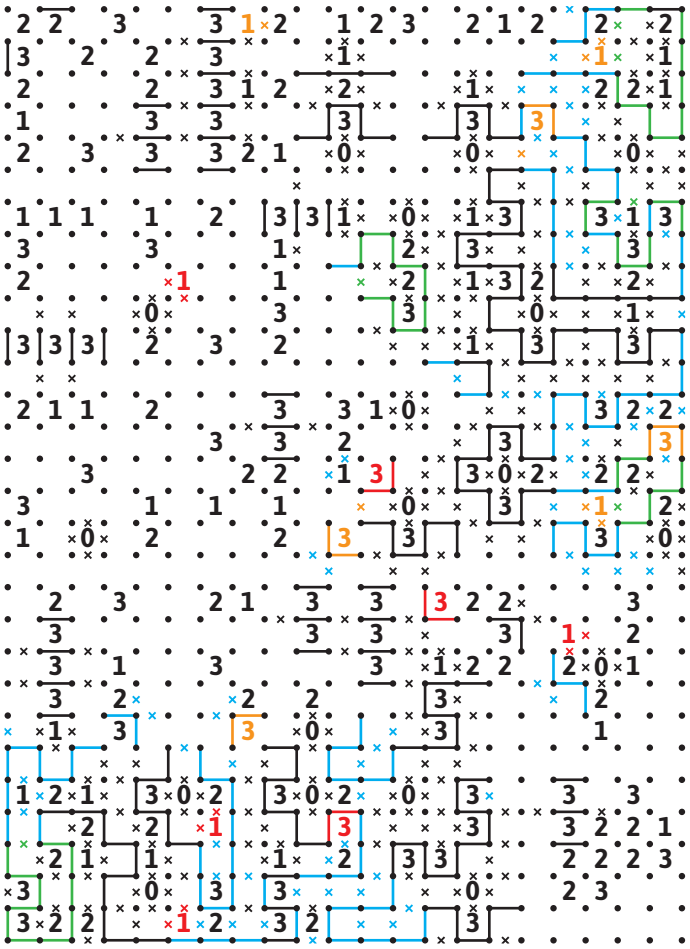


The next step is to 'chase the consequences' of these placements using two more simple rules:

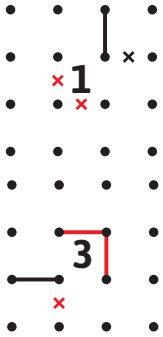
Clue completion: we can complete any clue where the number of lines around it equal that clue, or the number of x's around it equal 4 minus that clue.

Dot completion: any dot where two lines meet can be completed by placing x's on the other spots around it. Similarly, a dot with one line and two x's can be completed with another line, and a dot with three x's must have four x's around it.

There are times we will need actual deductions but for now this will suffice. By repeatedly applying these two rules we can progress quite far in the puzzle.



Now we must employ new pattern rules:



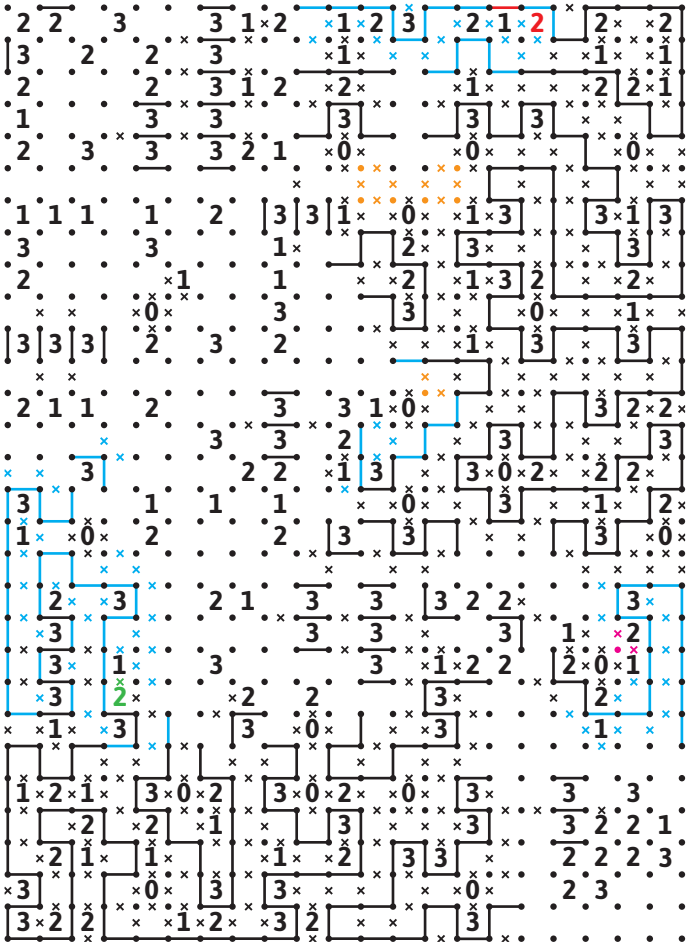
7) A line that meets a 1 clue along with an x must continue next to the 1, so the opposite two edges must be x's.

8) A line that meets a 3 clue can't form a corner there, so the opposite two edges must be lines.

Next, we can place an x wherever a line would close a loop. There are five such locations, shown in green. (The two in the top right may not be immediately obvious - if we put a line in either of the two open spots to the right of the orange 1, we're forced to close the loop, and if we put a line above the 1 surrounded by 3's, this forces the 3's around it to form a loop).

Second, the clues shown in red are similar to corner clues resulting in the red lines and x's.

Finally, we chase consequences again.

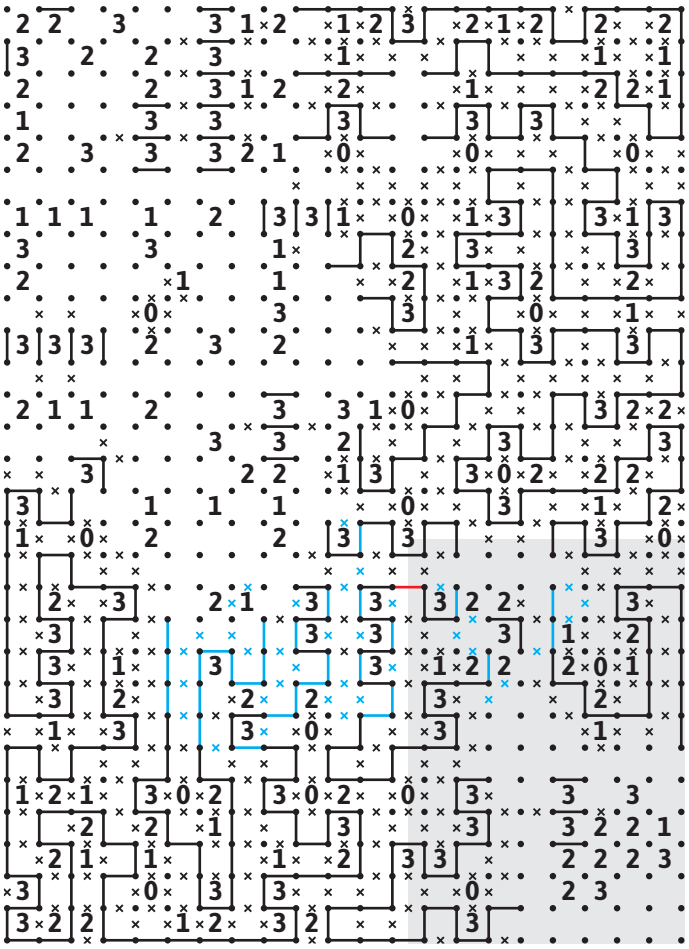


This is starting to get us somewhere. But we will need more and more sophisticated tactics, and progress will be more localised.

The red 2 can be treated as a corner clue, which gives us but a single extra line initially, but a lot more if we follow its implications.

Next, none of the orange dots can be completed with lines, because they would immediately force a small loop. The violet dot also leads to a loop, because completing it with lines would complete both the 2, 3, and 1 clues near it and leave a loop as the only option.

Finally look at the green 2. Note that if we put a line directly above it, we can't continue that line towards the top OR bottom. So it must be an x, and this gives a lot of extra lines and x's.

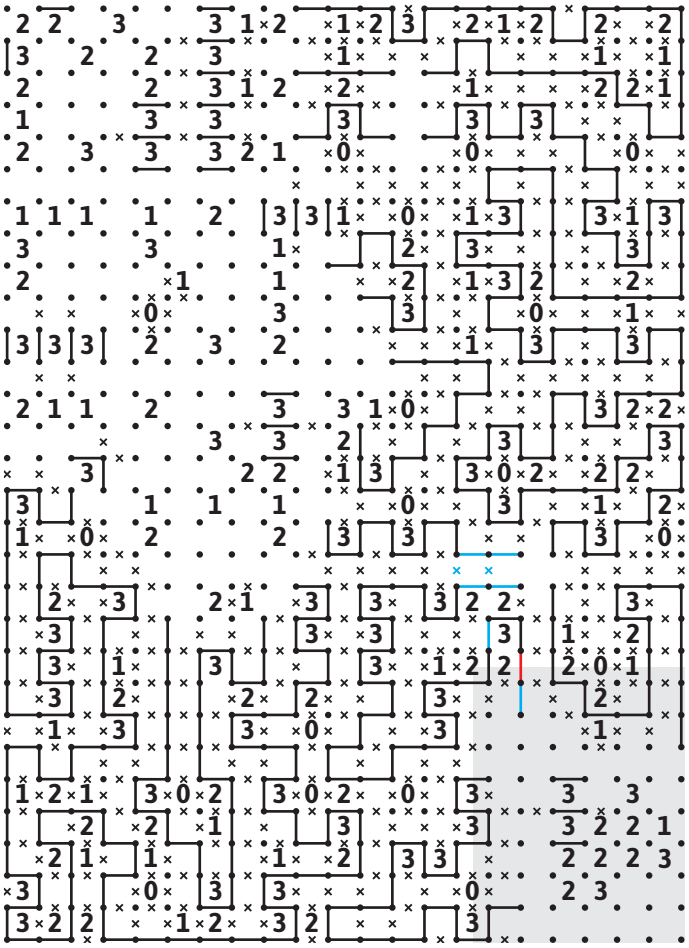


Besides repeatedly applying patterns and chains of logic, there is another technique we can use.

Since we must end up with a single closed loop, this means that if we draw any rectangle on the puzzle, an even number of lines must cross its border. If it were an odd number, the loop would not enter and exit the rectangle an equal number of times, which is impossible. This is equivalent to “there must be an even number of loose ends within any rectangle”

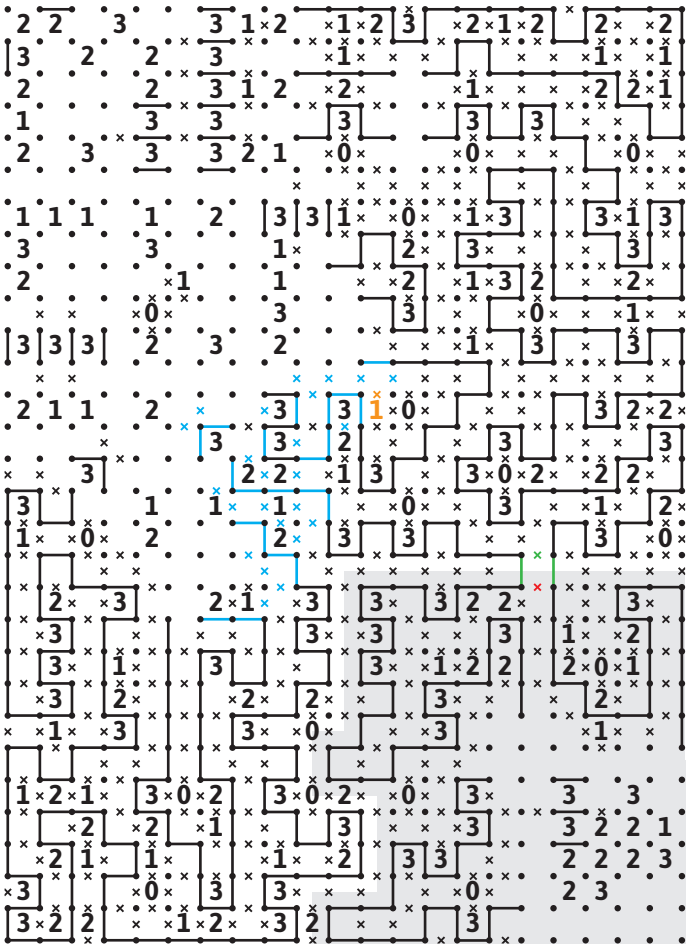
We can use this in the lower right of the puzzle. Consider the shaded rectangle. Do we continue the line to the left of the red 3 across its border or not? Currently nine lines cross the border - also there are seventeen loose ends within the rectangle. We must make both numbers even, and the only way is to continue the line to the left, out of the rectangle, as shown in red.

Consequences are shown in blue, as usual.



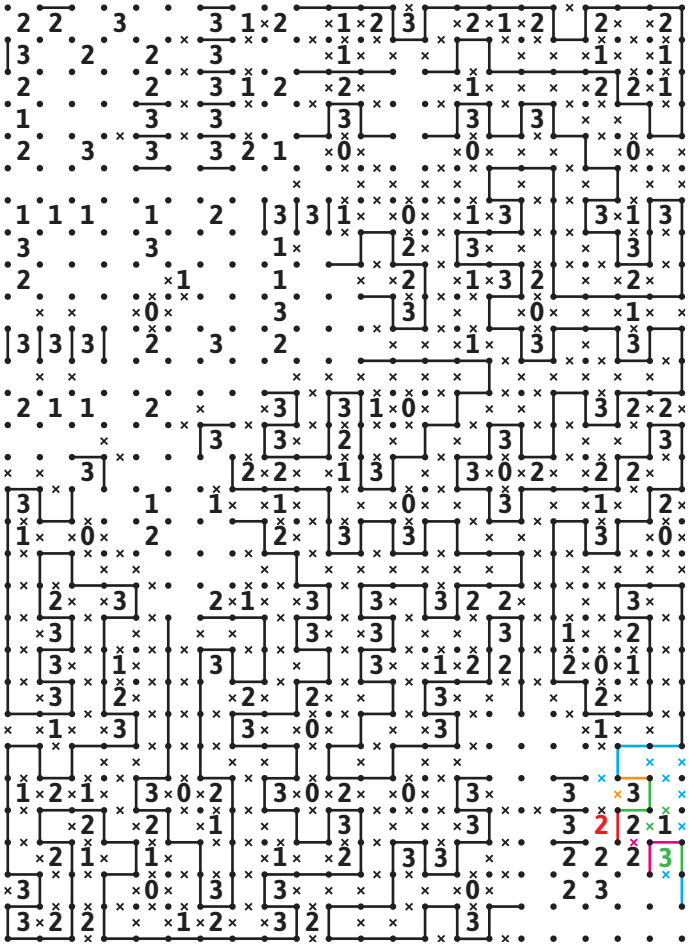
We can repeat this process with the smaller shaded rectangle and place the red line, and although it only gives us six more lines...

...luckily that is just enough to make another interesting deduction.



The shaded region is almost completely isolated from the rest of the puzzle. At its top there is one open spot left where there is neither an x nor a line. If we put a line there, the entire shaded region can at best form a single loop, but it can never connect to the rest of the puzzle. Therefore, we have to put an x there (in red). It gains us two important lines.

Next, we turn to the orange 1 in the center of the puzzle. If we were to put a line above it, this would force the 3 clue to be completed and result in a loop. So instead we put an x with a nice bit of progress to boot.



Now let's see if we can finish the bottom right corner. We begin with a few deductions.

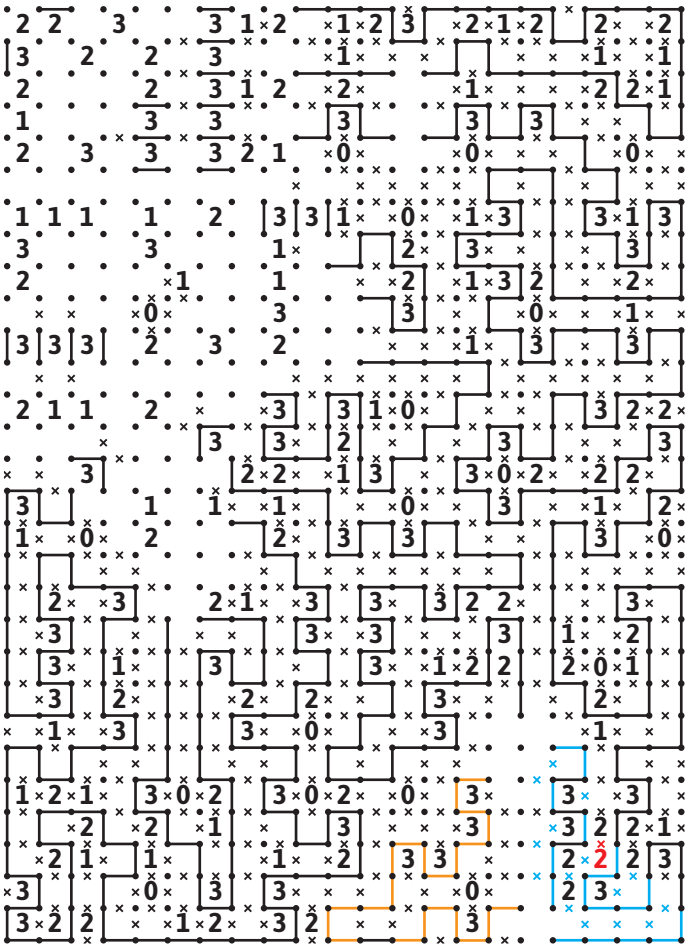
First, we can't put lines both to the left of and below the red 2, so there must be a line to its right.

Second, we can't put an x to the right of the green 3, or the line would be stuck above it, so it must be a line. This in turn forces x's to the top and left of the 1 above it, and this turns the 3 to its top left into a corner 3, and we can place two more lines.

Thanks to the line we already placed, we can now complete that 3.

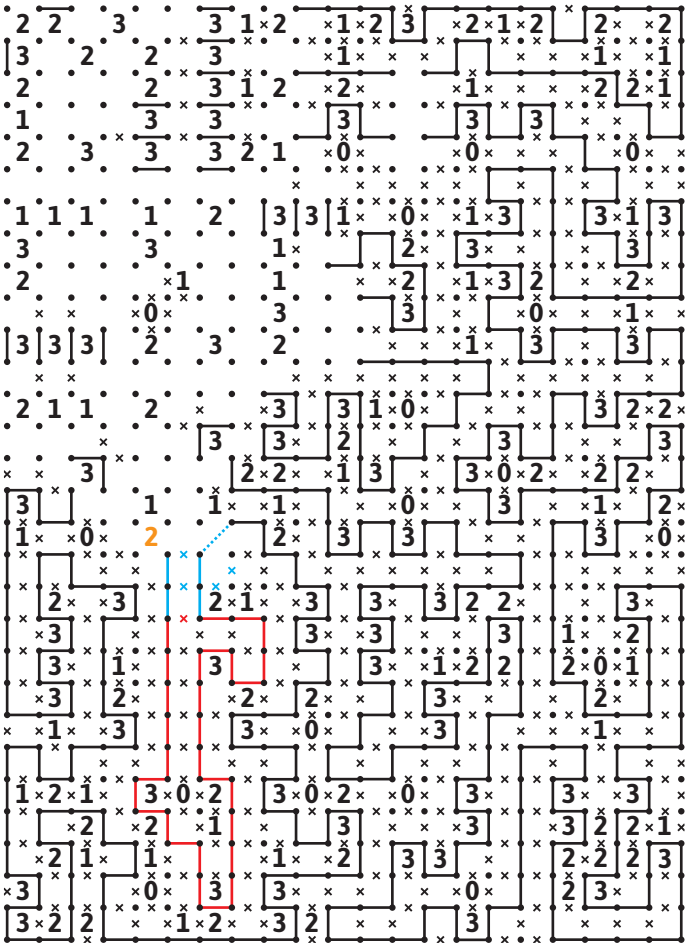
Since the 2 below it is also complete, we can return to the green 3 to complete it too.

Finally we follow the result to its logical conclusion.



If we put a line above the red 2, we would also have to put a line below it and its two neighbour 2's. This in turn conflicts with the 3 clue below them, so we must place an x instead. Now can progress quite far.

From this point (shown left) it is now obvious that the blue line along the bottom must continue upwards, not left, or we would end up with a loose end that can't connect anywhere. This in turn forces the orange line to its left to continue upwards as well. Along the way we need to avoid making an orange loop, and now the last bits resolve.



Speaking of loops, here's another one to avoid with an x. We can now extend this almost-loop with a few lines, another x, and again.

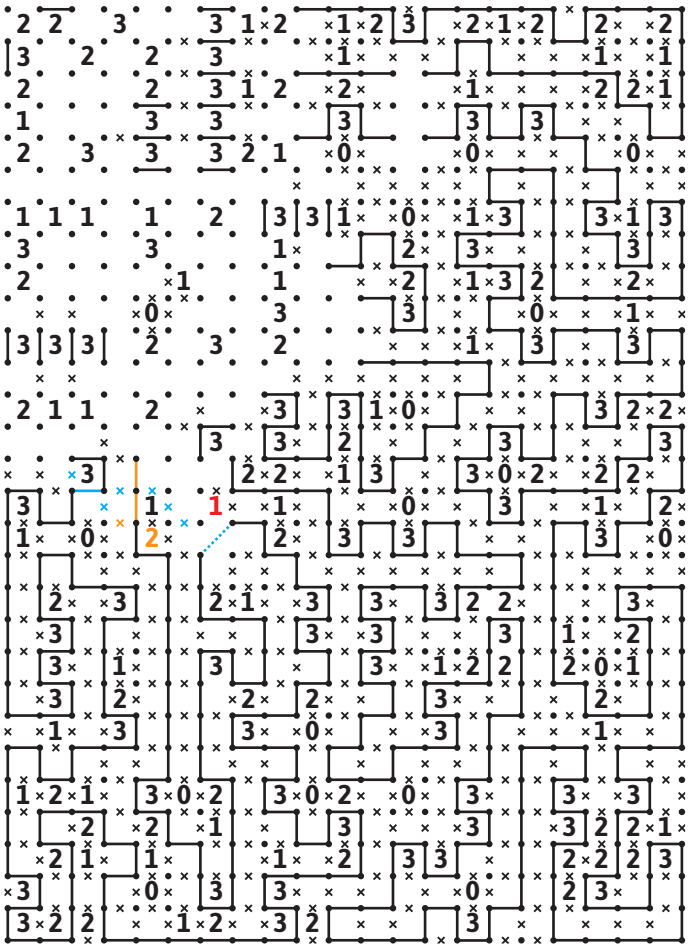
Check for yourself now that no matter how we continue the top right blue line, we will end up making the dotted connection.

This reveals that our red loop is surrounded by (and virtually connected to) a second loop!

Now consider the orange 2. It's effectively in a corner, so there are only two possible ways to put lines next to it: below it and to its left, or above it and to its right.

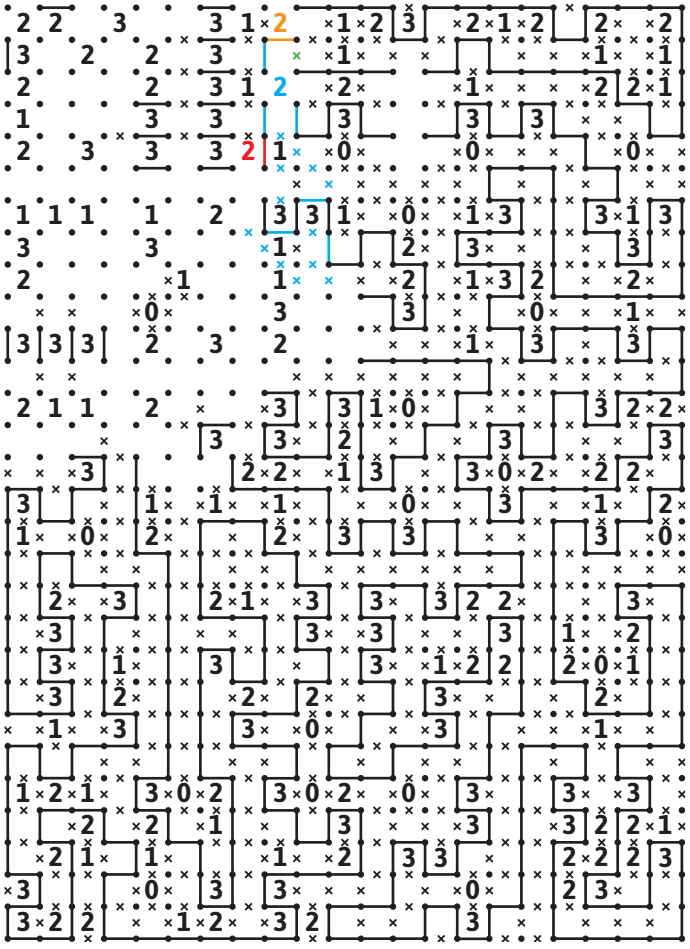
In the latter case, because we complete the 1 clue along the way, we would be forced to close the outer loop!

In other words, we must continue under and to the left of the orange 2.



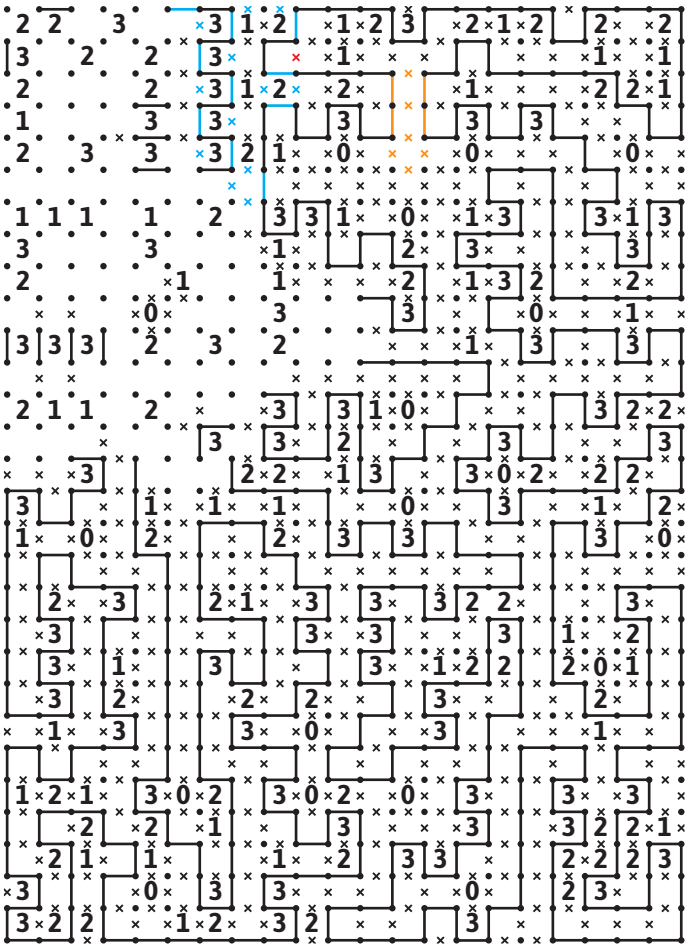
The path around the orange 2 can't continue to the left, or we would STILL close the outer loop, in fact we must avoid going towards the incomplete 3 clue at all cost. So we go up twice along the orange lines. Using the x's and lines that forces us to fill in, we are now ready to resolve the dotted line into the actual path.

Consider the red 1. It must have one line next to it, and no matter where we put it, that line must connect downward since there is a blue x preventing a path to the left. It follows that the dotted line must be resolved along the top and left.



Now we turn our attention elsewhere. First look at the red 2. We can't have a line both below it and to its left, so there has to be a line to its right. We can use the same reasoning to place another line below the orange 2.

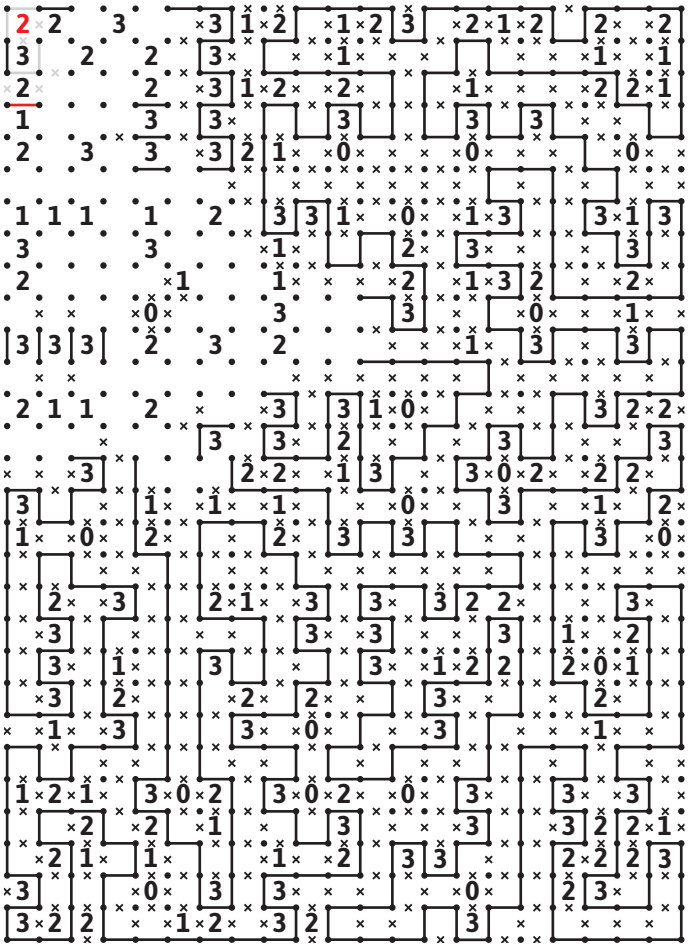
From these two we can fill in all the blue bits. Now look at the blue 2. It is now met with 4 lines, one in each corner. In other words, the loop can't form corners here, just like with the orange and red clues. This gives us an important x, shown in green.



From that one x (now marked in red) we can fill in all the blue stuff - we first complete the 2 above it, then move left, resolve the column of 3's, then back to the 2 that gave us the red x in the first place.

Now we have an almost-loop in the top right of the puzzle, leading to the orange resolution.

Almost there, I promise.

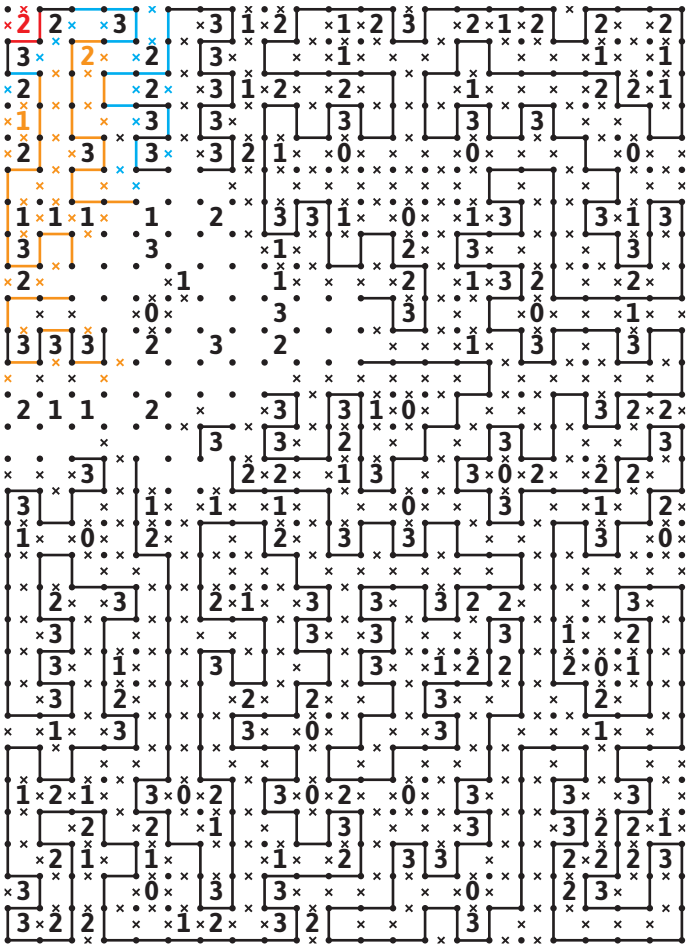


Consider the red 2 in the top left corner. (We could have started the whole solution here but that's the fun thing about slitherlink puzzles, you can attack them in many different ways.)

There's only two ways this clue can be completed - around the outside, or on the inside. The consequences of choosing the former are shown in light grey - first the 3 can only be completed in one way, which in turn forces us to complete the 2 clue below it with the red line.

But now we are stuck - the 1 clue below it is now also completed, and the red line becomes a dead end.

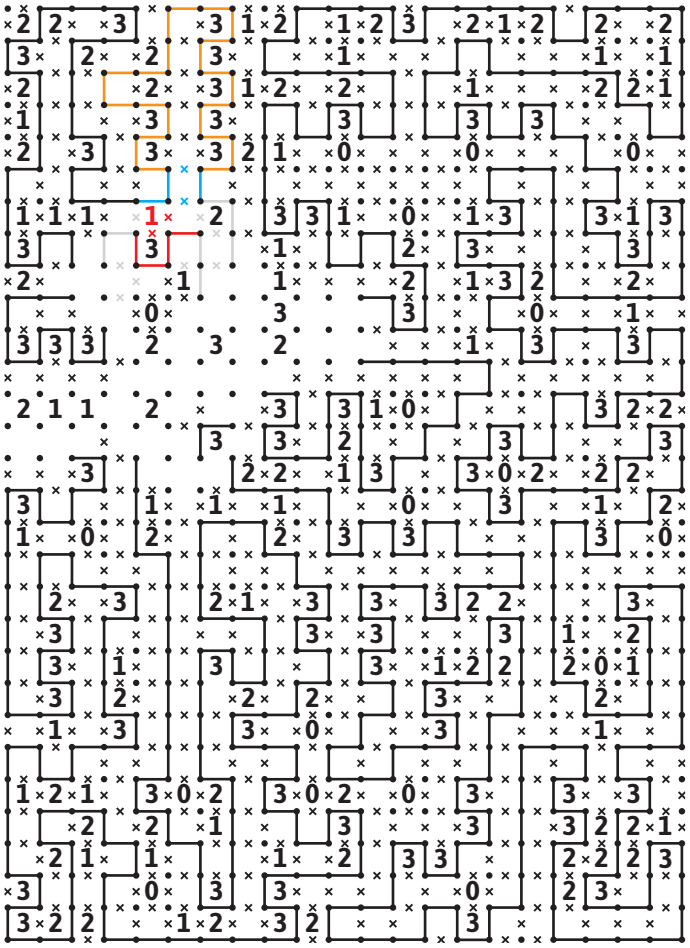
It follows that we must complete the red 2 on the inside instead.



If we apply the consequences of the red path we get the blue stuff. But we can go further. Consider the orange 2. It can only be completed in one way, by putting lines above it and to its left. The orange 1 is a 'corner case', so there must be x's above it and to its left.

Now, by faithfully completing one clue after another, we can complete the entire orange section - first we descend from the 2 above the orange 1, along the left edge of the puzzle, all the way down to the row of 3's, then we repeat the process descending from the orange 2.

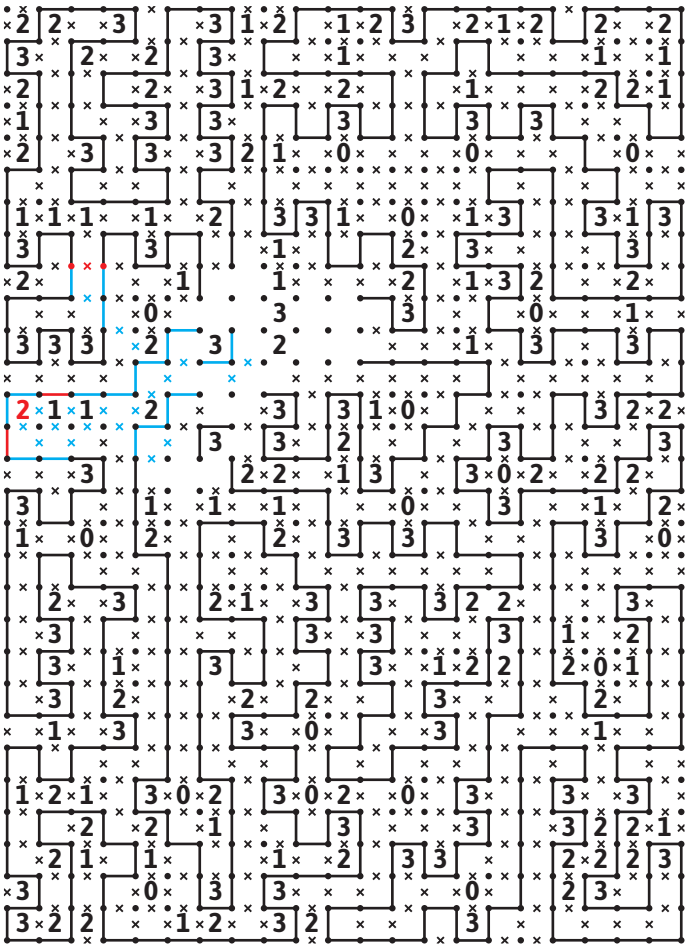
In fact we could continue a bit further, but in the interest of clarity let's stop here.



We're close to the end now. First, applying pattern 7, we can put two x's to the right of and below the red 1, which in turn completes the 3 below it.

Then we avoid closing the orange loop, shown in blue.

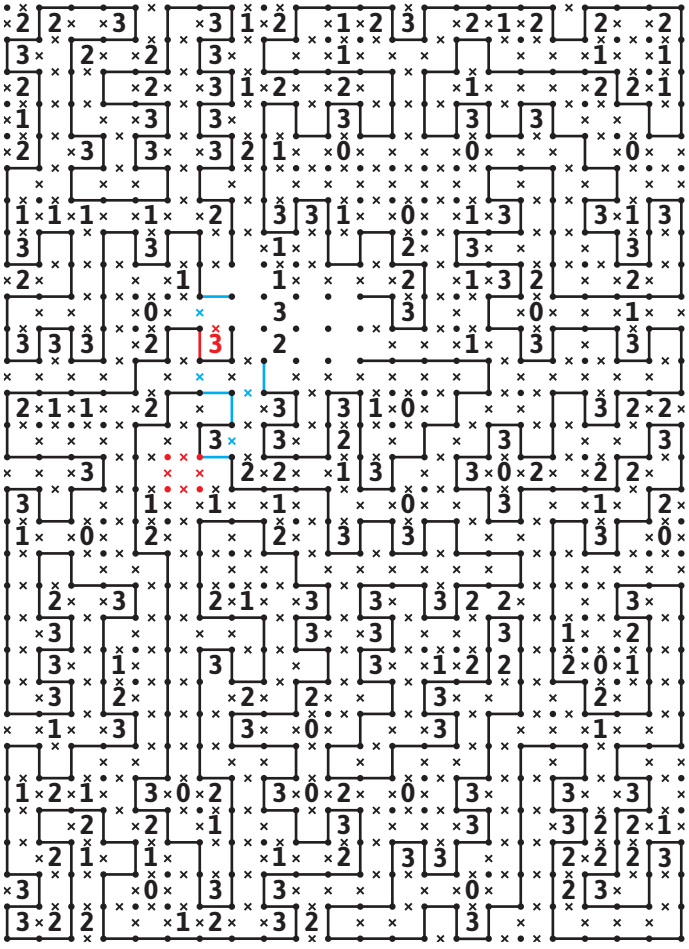
We then chase the consequences again, shown in light grey.



First, if we were to connect the red dots, this would cause a loop just below them, so we put the red x instead.

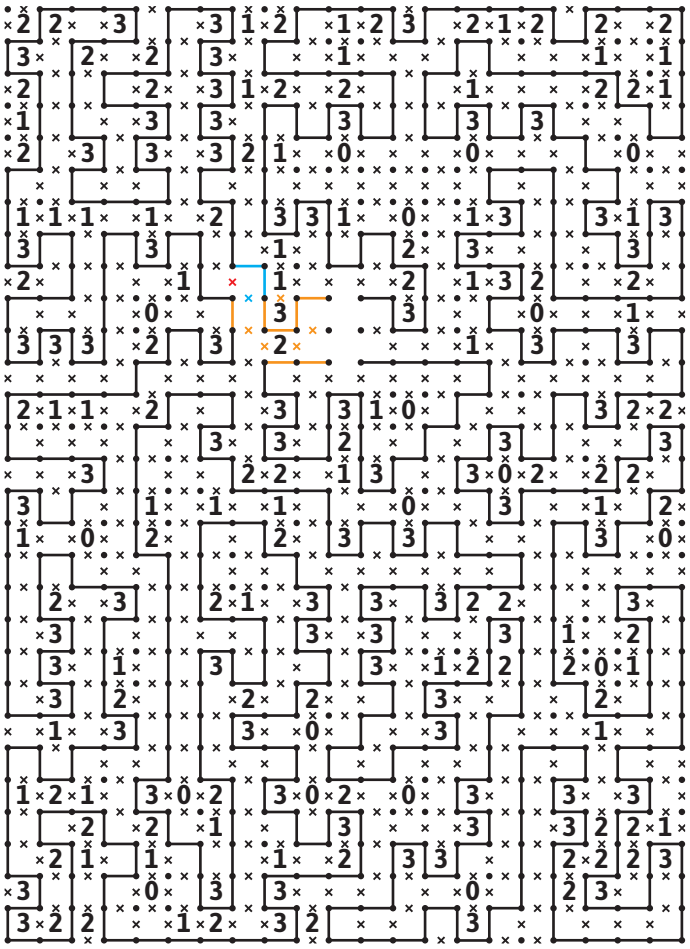
Second, the red 2 is effectively in a corner, so we can apply pattern 3 to get the two red lines and complete the 1 next to it.

These two placements are enough to net us all the blue progress, just by completing every clue and dot that we can.



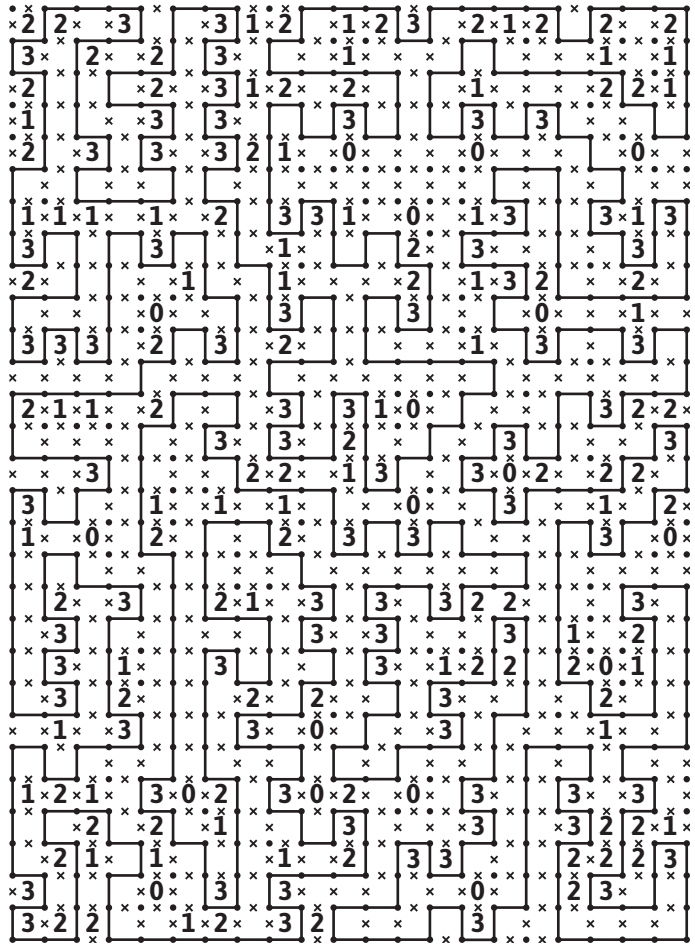
The red dots form a cul-de-sac - putting a line between any of them forces a loop, so x's it is. Speaking of loops, the red 3 can't be completed with a line above it, because that would create a large loop in the lower left of the puzzle. We complete it on its left instead.

Now we can progress almost to the end.



We can now complete the puzzle, by making sure we don't close the loop in the top left of the puzzle. This gives us the red x, and after avoiding a loop a second time, everything else now falls into place.

First we can progress by completing clues and dots, then, finally, the last remaining open space can be solved by once again avoiding a loop, this time in the center of the puzzle.



Here we have the completed solution.

Whew!

If you made it this far, I have a question for you. Was this solution booklet useful to you? Easy to follow? I tried to find a balance between clarity of the steps and enough progress per step to keep things going.

The actual process of solving a slitherlink flows quite naturally, it's hard to capture it in written form...

In any case, remember every Christmas card and solution booklet in the series can be found on my website - www.stack.nl/~angelo/